



## Engine Saturation Effect on Consensus of Decentralized Bi-Directional Nonlinear Self-Driving Vehicle Convoys

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### ABSTRACT

In this paper, the consensus of second-order nonlinear self-driving vehicle convoys (SDVCs) is studied. We assume that each self-driving vehicle (SDV) communicates only with one front and one rear SDVs. Each SDV's nonlinear dynamics consisting of the rolling resistance and the air drag force is a function of SDV's speed and is investigated in SDVC's modeling and consensus design. Since the speed is bounded, all vehicles' nonlinearities are also bounded. Due to engine saturation of each SDV, the control input is limited. We involve this limitation by introducing the arctan(.) function to control protocol. The inter-SDV's distances are assumed to be constant during motion. The distance tracking error associated with each SDV is defined as distance between it and the leading SDV. The error dynamics of the proposed SDVC is derived after applying the consensus law to each SDV. To prove the internal stability, the Lyapunov theorem is employed. We will prove that under this consensus algorithm, the SDVC will be internal stable. To validate the effectiveness of this method, a SDVC comprising a leading and 6 following SDVs will be studied. It will be verified that under the proposed consensus law, all the SDVs reach a unique consensus.

## 1.Introduction

In recent decades, we have perceived significant progress in controlling the motion of self-driving vehicles (SDVs) [1-3]. Self-driving vehicle convoys (SDVCs) have played a very immense role in creating and making intelligent traffic flows [4, 5]. The consensus as an important problem of SDVCs is investigated by many researchers [6-8]. We say that a SDVC achieves consensus if all SDVs reach a same speed and acceleration with a safe constant distance between consecutive SDVs [9].

The distance between SDVs can be fixed or variable. If constant, the length of the SDVC always remains constant, but if this distance is a function of the convoy speed, the length increases during acceleration and shortens during braking. If the interval between SDVs is always constant, the traffic capacity will be higher than when it is time-varying, although its practical implementation is more difficult [10, 11].

If the consensus has a unique solution, the SDVC is called internally stable. Moreover, if the distance error range does not increase among the

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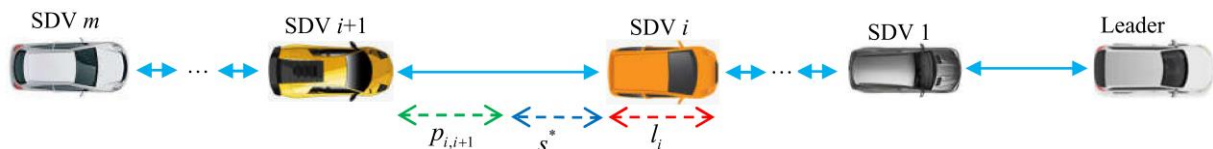
SDVs, the SDVC is called string stable [12]. According to how information is exchanged between SDVs, there are several communication structures in SDVCs. Directed structures such as centralized and decentralized predecessor following [13, 14], bi-directional [15] and multi-SDVs following [16].

Nonlinear dynamics and engine saturation have significant effects on stability of SDVCs. Due to several effects such as rolling resistance between tires and the road, power transmission structure and air resistant force the nonlinearities appear frequently in upper level dynamics of each SDV [17]. Nonlinearities may cause internal instability of SDVCs. Due to the limited speed of each SDV, all these nonlinear terms will be bounded. On the other hand, due to structural limitations of engine, the controller will be bounded. Few works have been carried out on internal stability of SDVCs with dynamical nonlinearities. A nonlinear consensus based on parameter identification is proposed in [18] for different topologies. A robust backstepping method to compensate the nonlinear dynamics is presented in [17]. Optimal consensus design in the presence of actuator delay and nonlinearities is performed in [19]. A nonlinear hierarchical model predictive approach to achieve the consensus and collision prevention of SDVCs is introduced in [20]. Effect of actuator fault on the stability of SDVCs is studied in [9] and [21]. The internal stability under input saturation, parameter uncertainty and time-varying distances is investigated in [22]. Adaptive robust finite-time consensus with unknown saturation and bound disturbance is presented in [23]. Neural network-based estimation design of uncertain SDVCs under

input saturation and nonlinear uncertainties is studied in [24]. The comfort and safety problems in the presence of input saturation, time delay and time-varying distances are proposed in [10]. In [25], the effects of actuator fault and saturation on convoy motion on uneven surfaces are investigated.

In the previous works, the consensus problem of second-order bi-directional (BD) decentralized SDVCs in the presence of nonlinearity and engine saturation has not been investigated. Therefore, we will solve the consensus problem of BD decentralized nonlinear second-order SDVCs in the presence of engine saturation. The motion of the leading SDV is known and all following SDVs' motion is described by second-order nonlinear differential models. The nonlinearities are caused by the rolling resistance and air forces. Therefore, all these terms are bounded by an  $\arctan(\cdot)$  function. Due to engine saturation, the control input of each SDV is limited. This limitation is modeled by the  $\arctan(\cdot)$  in the consensus law. All distances between SDVs are designed to be constant. The distance error of each following SDV with respect to leading SDV is defined and the dynamics of the closed-loop of the SDVC is obtained according to error dynamics. To obtain the consensus of the whole system, a Lyapunov function is defined. It will be proved that under the proposed consensus law, the BD decentralized SDVC with engine saturation will be internal stable. To verify the effectiveness of this method, numerical simulations are provided.

We arrange the remain of this article as below. In part 2, the problem is introduced and useful mathematical tools are presented. In part 3, the



**Figure 1.** A SDVC with bi-directional topology

consensus design procedure is presented. Part 4 provides the numerical studies and part 5 concludes the results and offers future guidelines.

## 1. System definition

Consider a SDVC of  $m$  SDVs and a leader with bi-directional topology as shown in figure 1. Leader motion is formulated by:

$$\ddot{p}_0(t) = \phi_0(t) \Rightarrow \begin{cases} \dot{p}_0(t) = q_0(t) \\ \dot{q}_0(t) = f_0(t) \end{cases} \quad (1)$$

where  $p_0(t)$ ,  $q_0(t)$  and  $f_0(t)$  are position, speed and the known function associated with the leader. Each of the SDV is formulated according to a second-order nonlinear equation as follows:

$$\begin{aligned} \ddot{p}_i(t) &= -(\alpha_{1,i} + \alpha_{2,i}q_i) / m_i - \alpha_{3,i}q_i^2 / m_i + u_i^* / m_i \\ \Rightarrow \begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = -(\alpha_{1,i} + \alpha_{2,i}q_i) / m_i - \alpha_{3,i}q_i^2 / m_i + u_i^* / m_i \end{cases} \end{aligned} \quad (2)$$

where  $\alpha_{1,i}$ ,  $\alpha_{2,i}$  are rolling resistance coefficients,  $m_i$  is the mass,  $u_i^*$  is consensus protocol and  $\alpha_{3,i}$  is a constant containing air drag coefficient and the geometry of the  $i$ -th SDV. By defining that  $f_i(q_i, t) = -(\alpha_{1,i} + \alpha_{2,i}q_i + \alpha_{3,i}q_i^2)$  and  $u_i = u_i^* / m_i$ , (2) will be rewritten as

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = f_i(t, q_i) + u_i \end{cases} \quad (3)$$

The consensus of the SDVC (1) and (3) is achieved if we have:

$$\begin{cases} \lim_{t \rightarrow \infty} |p_{i-1}(t) - p_i(t) - s^* - l_{i-1}| = 0 \\ \lim_{t \rightarrow \infty} |q_{i-1}(t) - q_i(t)| = 0 \end{cases}, \quad i = 1, 2, \dots, m \quad (4)$$

where  $s^*$  is the safe distance and  $l_k$  is the  $k$ -th SDV length, respectively.

**Lemma 1:** mean value theorem [26]. Suppose that  $h: [t_1, t_2] \rightarrow \mathbb{R}$  is a continuous function. There exists a constant  $t_1 < t^* < t_2$  such as:

$$\int_{t_1}^{t_2} h(\tau) d\tau = h(t^*)(t_2 - t_1) \quad (5)$$

**Assumption 1.** In practical implementations, the velocity of each SDV is bounded. Therefore, we can assume that

$$|f_i(t, q_i)| \leq c_i |\arctan(q_i)| \quad (6)$$

where  $c_i$  is a positive constant.

## 2. Consensus protocol design

To achieve the consensus in the presence of engine's saturation, the below consensus law is designed for each SDV.

$$u_i = \arctan(p_{i-1,i}) + \arctan(p_{i,i+1}) - \alpha_i \arctan(q_i) \quad (7)$$

where  $\alpha_i$  is a positive gain and:

$$\begin{aligned} p_{i-1,i} &= p_{i-1} - p_i - s^* - l_{i-1} \\ p_{i,i+1} &= p_{i+1} - p_i + s^* + l_i \end{aligned} \quad (8)$$

From (7), we can infer that:

$$|u_i(t)| \leq \pi \left( 1 + \frac{1}{2} \alpha_i \right) \quad (9)$$

By applying (7) to (3), the  $i$ -th SDV's closed-loop dynamics is obtained as below:

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = f_i(t, q_i) + \arctan(p_{i-1,i}) + \arctan(p_{i,i+1}) - \alpha_i \arctan(q_i) \end{cases} \quad (10)$$

We define that:

$$\begin{aligned} \xi_i &= \arctan(p_{i-1,i}) + \arctan(p_{i,i+1}), \xi = [\xi_1, \xi_2, \dots, \xi_m]^T \\ \alpha &= [\alpha_1, \alpha_2, \dots, \alpha_m]^T, \mathbf{p} = [p_1, p_2, \dots, p_m]^T, \mathbf{q} = [q_1, q_2, \dots, q_m]^T \\ \bar{q}_i &= \arctan(q_i), \bar{\mathbf{q}} = [\bar{q}_1, \bar{q}_2, \dots, \bar{q}_m]^T, \mathbf{f} = [f_1, f_2, \dots, f_m]^T \end{aligned}$$

Therefore, (10) is rewritten as:

$$\begin{cases} \dot{\mathbf{p}}(t) = \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) = \xi(t) - \alpha \bar{\mathbf{q}}(t) + \mathbf{f}(t) \end{cases} \quad (11)$$

Theorem 1 solves the consensus problem.

**Theorem 1.** If the parameter  $\alpha_i$  satisfies the following constraint, the SDVC described through (1) and (3) under the consensus law (7) and assumption 1 will obtain the consensus.

$$\alpha_i > c_i \quad (12)$$

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**Proof.** To prove, the below Lyapunov function is defined.

$$V = \sum_{i=1}^m \left( \int_0^{p_{i-1,i}} \arctan(\tau) d\tau + \int_0^{p_{i+1,i}} \arctan(\tau) d\tau \right) + \sum_{i=1}^m q_i^2 \quad (13)$$

According to lemma 1, there exists a positive value  $0 < \mu_i < p_{k,i}$  satisfying that  $\int_0^{p_{k,i}} \arctan(\tau) d\tau = \arctan(\mu_i) p_{k,i} > 0$  where  $k$  can be  $i-1$  or  $i+1$ .

Time differentiating of (13) along (11) yields

$$\begin{aligned} \dot{V} &= -2 \sum_{i=1}^m q_i (\arctan p_{i-1,i} + \arctan p_{i+1,i}) + 2 \sum_{i=1}^m q_i \dot{q}_i \\ &= -2 \mathbf{q}^T \dot{\xi} + 2 \mathbf{q}^T \dot{\mathbf{q}} \\ &= -2 \mathbf{q}^T (\dot{\mathbf{q}} + \mathbf{a}\bar{\mathbf{q}} - \mathbf{f}) + 2 \mathbf{q}^T \dot{\mathbf{q}} \\ &= -2 \mathbf{q}^T \dot{\mathbf{q}} - 2 \mathbf{q}^T \mathbf{a}\bar{\mathbf{q}} + 2 \mathbf{q}^T \mathbf{f} + 2 \mathbf{q}^T \dot{\mathbf{q}} \\ &= -2 \mathbf{q}^T \mathbf{a}\bar{\mathbf{q}} + 2 \mathbf{q}^T \mathbf{f} \\ &= -2 \sum_{i=1}^m \alpha_i q_i \arctan q_i + 2 \sum_{i=1}^m q_i f_i(t, q_i) \end{aligned} \quad (14)$$

According to assumption 1, one can write

$$\begin{aligned} \sum_{i=1}^m q_i f_i(t, q_i) &\leq \sum_{i=1}^m |q_i| \cdot |f_i(t, q_i)| = \\ &= \sum_{i=1}^m c_i q_i \arctan q_i \end{aligned} \quad (15)$$

Therefore,

$$\begin{aligned} \dot{V} &\leq -2 \sum_{i=1}^m \alpha_i q_i \arctan q_i + 2 \sum_{i=1}^m c_i q_i \arctan q_i \\ &= -2 \sum_{i=1}^m (\alpha_i - c_i) q_i \arctan q_i \end{aligned} \quad (16)$$

and under (12), we have  $\dot{V} \leq 0$ .

According to (16), it is inferred that when  $\dot{V} = 0$  we have  $q_i = 0$ . From (10), we have

$$f_i(t, q_i) + \arctan(p_{i-1,i}) + \arctan(p_{i+1,i}) = 0 \quad (17)$$

From assumption 1, we have  $|f_i(t, q_i)| \leq c_i |\arctan(q_i)| = 0$  and

$$\begin{aligned} f_i(t, q_i) = 0 \Rightarrow \\ \arctan(p_{i-1,i}) + \arctan(p_{i+1,i}) = 0 \end{aligned} \quad (18)$$

So that,

$$\begin{aligned} \sum_{i=1}^m (p_i - s^* - l_{i-1}) \arctan(p_{i-1,i}) &= 0, \\ \sum_{i=1}^m p_{i-1,i} \arctan(p_{i-1,i}) &= 0 \end{aligned} \quad (19)$$

By subtracting the above equations and knowing that, the network topology is bi-directional and therefore, symmetric, we will have:

$$\sum_{i=1}^m p_{i-1,i} \arctan(p_{i-1,i}) = 0 \quad (20)$$

We know that for  $\mathcal{G} \neq 0$ :  $\mathcal{G} \arctan(\mathcal{G}) > 0$ . Therefore, from (20) we infer that  $p_{i-1,i} = 0$  and according to (4) the consensus is achieved and the proof is complete.

**Remark 1.** Theorem 1 is presented for bi-directional network topology. This approach can be applied to all symmetric networks.

In the following, we present a comparison for a case where engine saturation is not considered. For this case, the consensus protocol (7) is redesigned as follows:

$$u_i = p_{i-1,i} + p_{i+1,i} - \bar{c}_i q_i \quad (21)$$

where  $\bar{c}_i$  is a positive gain. In the case that the input has no saturation, assumption 1 is modified as follows.

**Assumption 2.** Without considering saturation, the nonlinear function  $f_i(t, q_i)$  is bounded by

$$|f_i(t, q_i)| \leq c_i^* |q_i| \quad (22)$$

where  $c_i^*$  is a positive constant. Under (21), (10) will be as below:

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = f_i(t, q_i) + p_{i-1,i} + p_{i+1,i} - \bar{c}_i q_i \end{cases} \quad (23)$$

By defining  $\bar{\xi}_i = p_{i-1,i} + p_{i+1,i}$ ,  $\bar{\xi} = [\bar{\xi}_1, \bar{\xi}_2, \dots, \bar{\xi}_m]^T$  and  $\bar{\mathbf{C}} = \text{diag}(\bar{c}_1, \bar{c}_2, \dots, \bar{c}_m)$ , (11) will be as follows

$$\begin{cases} \dot{\mathbf{p}}(t) = \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) = \bar{\xi}(t) - \bar{\mathbf{C}}\mathbf{q}(t) + \mathbf{f}(t) \end{cases} \quad (24)$$

Now, we modify theorem 1 as follows.

**Theorem 2.** The SDVC described through (1) and (3) with unsaturated engine's input, under the consensus law (21) and assumption 2 will reach to consensus if we have:

$$\bar{c}_i > c_i^* \quad (25)$$

**Proof.** To prove the consensus problem, the following Lyapunov function is defined.

$$V = \frac{1}{2} (p_{i-1,i}^2 + p_{i+1,i}^2) + \sum_{i=1}^m q_i^2 \quad (26)$$

Differentiating (26) yields

$$\begin{aligned} \dot{V} = & \sum_{i=1}^m [(q_{i-1} - q_i) p_{i-1,i} + (q_i - q_{i+1}) p_{i+1,i}] + \\ & + 2 \sum_{i=1}^m \dot{q}_i q_i \end{aligned} \quad (27)$$

By simplifying, (27) can be written in the following compact form

$$\begin{aligned} \dot{V} = & -2\mathbf{q}^T \bar{\xi} + 2\mathbf{q}^T \dot{\mathbf{q}} \\ = & -2\mathbf{q}^T (\dot{\mathbf{q}} + \bar{\mathbf{C}}\mathbf{q} - \mathbf{f}) + 2\mathbf{q}^T \dot{\mathbf{q}} \\ = & -2\mathbf{q}^T \bar{\mathbf{C}}\mathbf{q} + 2\mathbf{q}^T \mathbf{f} = -2 \sum_{i=1}^m \bar{c}_i q_i^2 + 2 \sum_{i=1}^m q_i f_i(t, q_i) \end{aligned} \quad (28)$$

According to assumption 2, one can write

$$\sum_{i=1}^m q_i f_i(t, q_i) \leq \sum_{i=1}^m |q_i| \cdot |f_i(t, q_i)| \leq \sum_{i=1}^m c_i^* q_i^2 \quad (29)$$

Consequently,

$$\dot{V} \leq -2 \sum_{i=1}^m \bar{c}_i q_i^2 + 2 \sum_{i=1}^m c_i^* q_i^2 \leq -2 \sum_{i=1}^m (\bar{c}_i - c_i^*) q_i^2 \leq 0 \quad (30)$$

According to (30), it is inferred that when  $\dot{V} = 0$  we have  $q_i = 0$ . From (23), we have

$$f_i(t, q_i) + p_{i-1,i} + p_{i+1,i} = 0 \quad (31)$$

From assumption 2, we have  $|f_i(t, q_i)| \leq c_i^* |q_i| = 0$ . So that

$$f_i(t, q_i) = 0 \Rightarrow p_{i-1,i} + p_{i+1,i} = 0 \quad (32)$$

Accordingly,

$$\sum_{i=1}^m p_{i-1,i} p_{i+1,i} = 0 \quad (33)$$

The network topology is symmetric. Therefore,

$$\begin{aligned} \sum_{i=1}^m p_{i-1,i}^2 &= \sum_{i=1}^m p_{i-1,i} p_{i-1} - \sum_{i=1}^m p_{i-1,i} p_i = \\ &= 2 \sum_{i=1}^m p_{i-1,i} p_{i-1} = 0 \end{aligned} \quad (34)$$

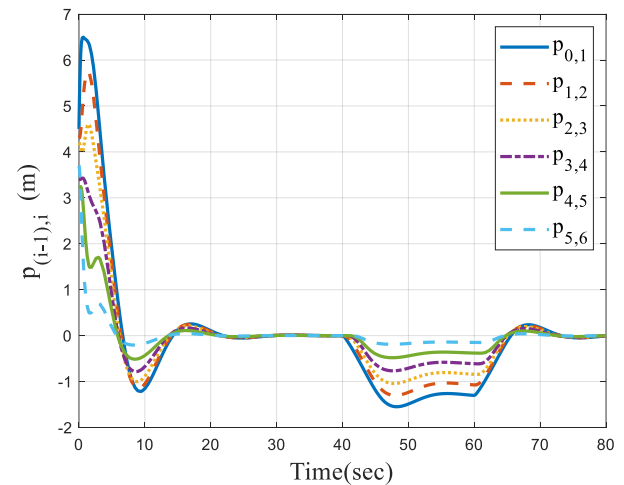
which indicates that  $p_{i-1,i}$ ,  $i = 1, 2, \dots, m$  and the consensus has a unique solution.

**Remark 2.** It should be noted that the coefficients  $\alpha_{1,i}$ ,  $\alpha_{2,i}$  and  $\alpha_{3,i}$  are small. When they are divided by  $m_i$ , since the velocity is bounded, the resultant nonlinear term  $f_i(q_i, t)$  will be smaller and therefore, the assumptions (6) and (22) are reasonable.

### 3. Verification study

To verify the performance of the proposed consensus algorithm in theorem 1, a convoy comprising 7 SDVs is considered. We perform the simulation for two braking and accelerating maneuvers. The constants are supposed as:

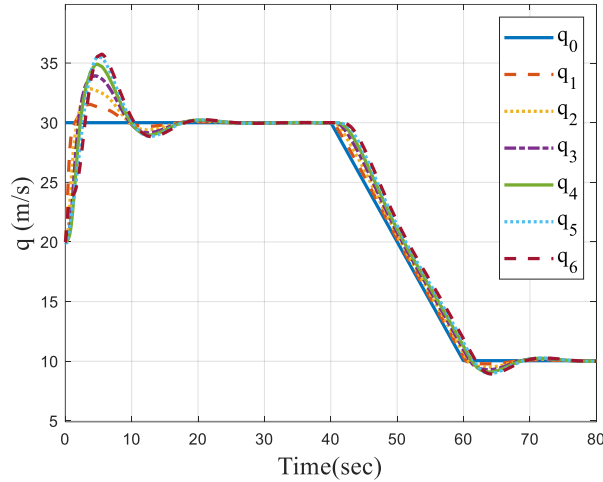
$$\begin{aligned} s^* &= 5m, \alpha_i = 4.6, \bar{c}_i = 4.1, m_1 = 1400kg, m_2 = 1500kg, \\ m_3 &= 1350kg, m_4 = 1450kg, m_5 = 1410kg, m_6 = 1440kg, \\ l_1 &= 3.5m, l_2 = 3.8m, l_3 = 4.2m, l_4 = 4.4m, l_5 = 4.3m, \\ l_6 &= 3.8m. \end{aligned}$$



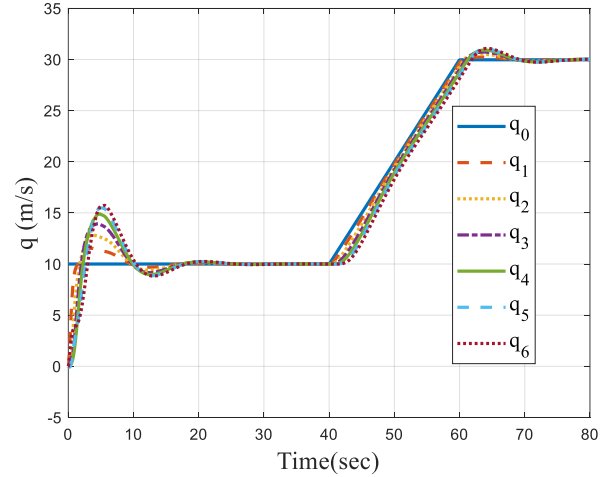
**Figure 2:**  $p_{i-1,i}$  for the convoy: braking maneuver

Figures 2 and 5 illustrate the distance error of the SDVC in braking and accelerating maneuvers, respectively. As these figures show, all distance tracking errors tend to zero specifying internal stability in both acceleration and braking maneuvers. Figures 3 and 6 depict the velocity and figures 4 and 7 show the acceleration of SDVs in braking and accelerating maneuvers, respectively. Since the SDVC is internal stable, the following SDVs track the speed and acceleration of leading SDV.

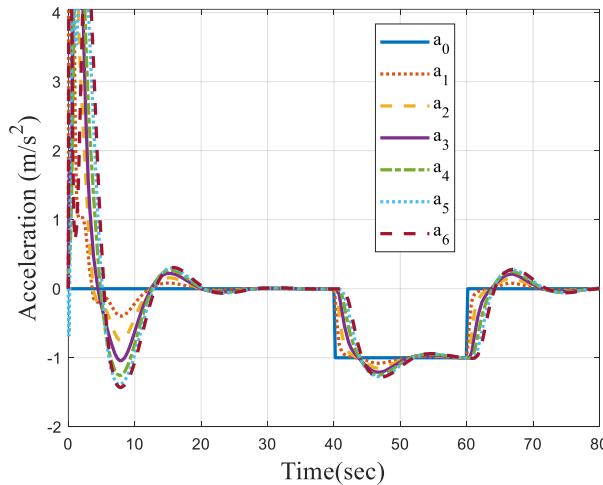
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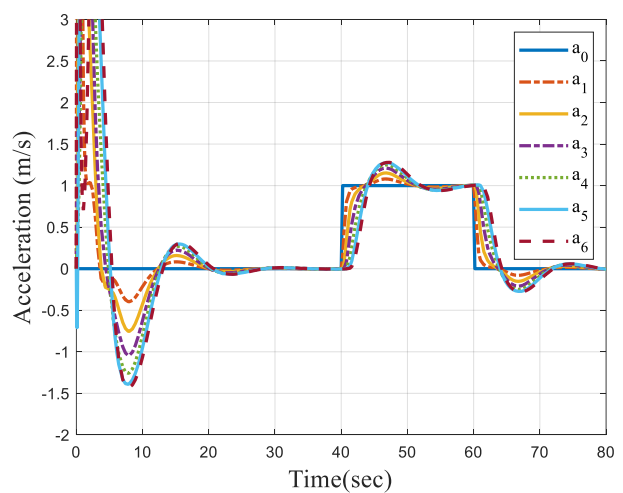
**Figure 3:** Speed of the convoy: braking maneuver



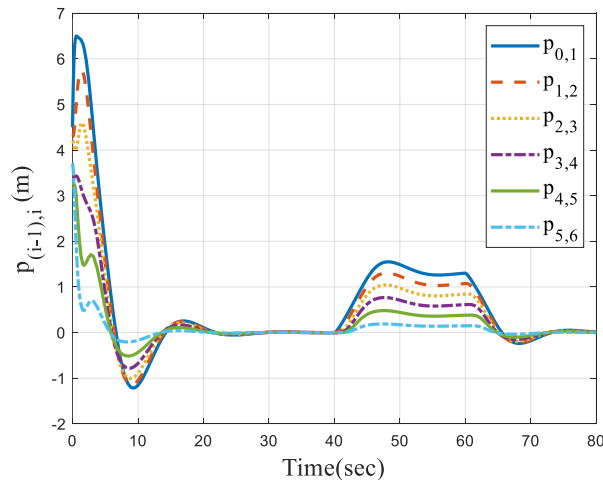
**Figure 6:** Speed of the convoy: accelerating maneuver



**Figure 4:** Acceleration of the convoy: braking maneuver



**Figure 7:** Acceleration of the convoy: accelerating maneuver



**Figure 5:**  $p_{i-1,i}$  for the convoy: braking maneuver

## 4. Conclusions

The consensus of second-order nonlinear SDVCs with bi-directional topology in the presence of engine saturation was studied. Each SDV's nonlinear dynamics consisting of the rolling resistance and the air drag force is a function of SDV's speed. Due to engine saturation, the control input is limited. We involved this limitation by introducing the  $\arctan(\cdot)$  function to control protocol. The error dynamics of the proposed SDVC was derived after applying the consensus law to each SDV. To prove the internal stability, the second Lyapunov theorem was employed. It was shown that under this consensus algorithm, the SDVC is internal stable. To verify the effectiveness of this method,

a SDVC comprising a leading and 6 following SDVs was studied. The obtained results showed the merits of the proposed method. For future works, the engine time constant can be added to the presented approach of this paper.

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