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Modal shape identification of the vibration data of bridge dynamic test using fuzzy clustering

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Abstract

The transmission of vibration to the adjacent buildings due to the traffic passing over Ghalemorghi Bridge in Tehran, has caused disruption to the inhabitants of the buildings. In order to investigate the vibration condition of the bridge, a dynamic loading test was performed and the output was analyzed using a non-parametric modal identification approach. In this article, the non-parametric modal identification algorithm from output measurement has been interpreted corresponding the clustering algorithm by subset relation with complex exponential basis functions on basis of soft computing paradigm. Moreover, the free vibration of bridge acceleration under the loading of a passing truck is processed by this identification method, and the acceleration vibration data integration in frequency region is performed by fuzzy clustering in order to determine the modal shape. Finally, vibration of the bridge elastic bearings and the similarity of the deck natural frequency to the vibration frequency of "Deck-Elastic bearing" system are recognized as two major problems of the bridge.

Keywords: Traffic induced vibration; Vibration data fusion; Modal shape identification; Fuzzy clustering; Modal identification from output measurement; Bridge loading test; Condition evaluation

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1. Introduction

Vibration condition evaluation is one of the non-destructive structural evaluation methods of bridges which are performed in form of dynamic test. The dynamic test is carried out in different ways. From which the test with shaker, impact test, specific vehicle passing test, and environmental vibration (due to wind or traffic) test can be mentioned (Farrar, Charles, & Worden, 2007). In this article, the test under passing of a specific vehicle is used.

In order to evaluate the condition, one should pass through the measurement space, and reach the decision making space by gathering the data to the property space, and give statement about the condition of the structure. The property space which is usually considered in dynamic tests are modal properties of the system (natural frequency, damping and modal shape) which are mostly resulted by assuming separate time and location, and existence of classic mechanics principles in description of system dynamics and assumption of being linear and invariable with time. Passing from property space to decision making space is completed by relating the modal properties to mechanical and geometrical properties of the system and give statement about the condition of the structure. Although over a century has passed from application of dynamics tests in structure condition evaluation, but these methods have not become common in huge civil structures area.

Classic data processing methods in these tests are based on parametric or non-parametric identification of the system, and on the basis of statistics and probability theory, and the modern methods are based on statistical learning theory (Mohammadzadeh, 2004). In this article, the non-parametric modal identification method is considered on basis of soft computing paradigm, and this method is interpreted corresponding a clustering algorithm by subset relation with complex exponential basis functions. Besides, a method for integrating the vibration data in frequency space is presented in order to determine the modal shape by fuzzy clustering.

Hence, in the second part of the article, the modal identification is described, the method of problem solving from the output is clarified and interpreted on basis of soft computation. In the third part, instrumentation and dynamic test of Ghalemorghi Bridge is presented. In the fourth part, the non-parametric identification of bridge modal parameters are performed by processing the free vibration of bride acceleration under the loading of a passing truck and the modal shapes are derived by fuzzy clustering. In the fifth part, the reasons of vibration transmission to the adjacent buildings due to the traffic passing over the bridge are presented in form of conclusion.

2. Modal identification and its interpretation on basis of soft computation

Modal identification is estimating the modal parameters of the structural system from measured input and output data. The modal parameters include complex-valued modal frequency ($\lambda_r = \sigma_r + j\omega_r$), modal vectors, modal scale (modal mass M_r), modal participation vector, and the other residue vectors. σ_r is damping and ω_r is the natural damping frequency. The modal participation vector shows that how each modal vector is excited by reference points of measured data. Combination of modal participation vector and modal shape vector for each mode, gives the residue matrix of that mode. In modal analysis from output, since the system input is not measured, the resulted modal shapes are not scaled and hence, the modal mass of each mode is not achievable. In classic way, the modal test is performed by simultaneous measurement of input and output. But, technical and economical limitations particularly in civil structures have brought modal test with output measurement into attention (Peeters & Ventura, 2003). Some assumptions are considered on input loading in these kinds of tests in order to analyze the data. For instance, in natural excitation techniques (NExT), the input is assumed as white noise (James, Carne, & Lauffer, 1995). This assumption is considered in civil structures vibration due to environmental vibration such as wind, wave, and passing of vehicles. Considering this assumption, the output signal function and the functions of its other correlations are considered as functions of system impact. For instance, the correlation function between the *i* and *j* response signal is considered as the response of *i* location due to the impact in *j* location. When the assumption of the white noise input is not satisfied, application of random decrement signature can be useful. This index is the correlation function with initial condition and has been considered in processing the vibration data due to passing of vehicles on road bridges (Asmussen, Brincker, & Ibrahim, 1999).

Various algorithms have been presented for estimation of modal parameters. All the mentioned methods which are some in time and frequency domains can be displayed in form of unified matrix polynomial algorithm (UMPA) context (Allemang & Brown, 1998).

Different algorithms have similar initial point. All the algorithms are derived from a linear quadratic equation of system with constant coefficients. This principal equation is based on various assumptions such as linearity, invariance by time, observability, and reciprocity (Allemang & Brown, 1998). The current approach in modal identification is using numerical techniques in separating the share of different vibration modes in measured data. In relation (1), $2N$ is the number of modes, N_0 is the number of output degree of freedoms, and N_i is the number of input degree of freedoms:

$$[H(\omega)]_{N_0 \times N_i} = \sum_{r=1}^{N} \frac{[A_r]_{N_0 \times N_i}}{j\omega - \lambda_r} + \frac{[A_r^*]_{N_0 \times N_i}}{j\omega - \lambda_r^*} \quad (1)$$

Relation (1) is nonlinear in respect to modal frequencies. When the modal frequencies are determined, the problem becomes linear in respect to the remaining modal parameters ($[A_r]$). Hence, in modal parameters identification algorithms, the modal frequencies are calculated at first, then the modal shapes, modal scale vectors and the residue are calculated.

In modal identification from output measurements, the system input in deterministic scence, is zero (PPT, FDD, LSCE, ITD, ERA methods), and is white noise in random scence (SSI method). Hence, the system free vibration should be considered to identify modal parameters, both the free vibration measured directly and the free vibration extracted from criteria such as random decrement signature measured indirectly.

The perceiving core of modal parameters context is seeking an order in the data. The man observes the order in nature and applies it to foresee the events. For instance, when we come across a sequence of numbers such as 1010101?, the next number is predicted to be 0. This order may be in various scales. If the interval between the occurrence of the mentioned sequence is Δt, its order would have period of $2\Delta t$.

The order may be observed in other scales or periods. Perceiving the order in observations is performed in form of sine and cosine and form of complex exponential basis functions, in general. Displaying in form of complex exponential will consider the damping effect in natural observations. The mathematical tool for searching a damping order in continuous space is Laplace transform, and for non-damping order in continuous space is Fourier transform. The mentioned transforms in discrete space will change into spiral *Z* -Transform, and *Z*-Transform around unit circle (discrete Fourier transform), respectively.

In *Z*-Transform around unit circle (discrete Fourier transform), similarity of signal with $e^{-i\frac{2\pi j}{N}t}$ for *j* from 0 to $N - 1$ is evaluated by similarity criterion, in which *N* is the length of main signal. It should be noted that the similarity criterion is appropriate for stationary processes and the Aliasing and Leakage phenomena should be considered when using it.

In other words, the problem of system identification will change to estimating the system free vibration response in form of the sum of modal shapes which are as complex exponential functions (periodic and reductive). This is seeking the similarity relation in data.

Generally, a relation with Reflexivity, Symmetry, and Transitivity properties among the relations of ordered pairs, is called a Similarity relation. This relation is called Equivalence relation in classic set theory. As we know, each equivalence relation displays a partition and each partition defines an equivalence relation. Hence, in mathematical point of view, clustering is performed by finding an equivalence relation, and the pattern governing the data is discovered. Two equivalent samples will be in one cluster and two non-equivalent samples will be in separate clusters.

Different explanations of similarity relation are presented in fuzzy sets theory, from which Similarity measure (suggested by Zadeh), Likeness relation (suggested by Rusppini), and Indistinguishability relation (suggested by Valverde and Mantaras) can be mentioned. By defining the similarity relation, we can present a definition for distance (non-similarity), in which similarity is related to distance by De Morgan law. Hence, in mathematical point of view, finding an equivalence relation (in classic sets theory) or a similarity relation/likeness relation/indistinguishability relation (in fuzzy sets theory) in the data, is equal to identification of similar patterns in the data (Ataei, Aghakouchak, Marefat, & Mohammadzadeh, 2005).

Considering the explanations above, the non-parametric modal identification method from output on basis of soft computing paradigm is interpreted corresponding a clustering by subset relation with complex exponential basis functions in which the similarity of the main signal with the basic signals is done by internal Multiplication, and the decision making towards similarity is done by integrating the average (mean). Linear clustering, the assumption of similar speed of wave propagation, averaging in similarity decision makings, assumption of existence of real mode (instead of complex mode) and leakage phenomenon are considered as limitations of this method.

3. Dynamic test of Ghalemorghi Bridge

The Ghalemorghi Bridge (Fig. 1), located at the end of Ghalemorghi Street in Tehran, provides an overpass for a four lane road over the double railway track of south. The 225-m long bridge consists of seven pans (30 + 30) + (30 + 45 + 30) + (30 + 30), and the deck has joints on the third support from both sides. The bridge is located in horizontal curve with radius of 140 m. The bridge deck is consisted of four composite box beams with height of 1.8 m, and distance of 4.4 m. The width of bridge is 18 m. Each beam is located on an elastic circular bearing (Track and Substructure Office of Iran Railway, 2007).

Full-size image (49K)

Fig. 1.

The 45-m span of Ghalemorghi Bridge.

In vibration evaluation test of the mentioned bridge, the third and fourth span of the (30 + 45 + 30) span was instrumented. The length of this span is 101 m in first lane of traffic and 112 m in the fourth one, and the first and second span were instrumented and all the three spans were loaded.

Thirty-four acceleration sensors were used in the bridge test. The measuring tools were installed in five sections of the bridge. Three sections on the piers and two sections in the middle of third span (30 m) and fourth span (45 m) were selected (Fig. 2a, Fig. 2b and Fig. 2c).

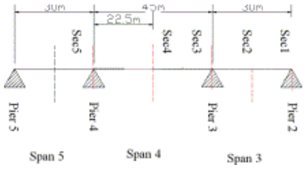

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Fig. 2a.

The instrumentation on the third span of the bridge.

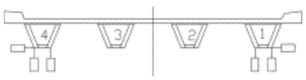

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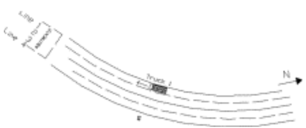
Fig. 2b.

The instrumentation on five cross-sections with acceleration sensors.



Full-size image (14K)

Fig. 2c.

The plan of truck movement over the bridge with different speeds.

The dynamic test were performed by passing a three-axle truck with axial load of 28 tones on all the four lanes in both direction with speeds of 5 and 25 km/h, which will be 16 sets of tests (Fig. 3).

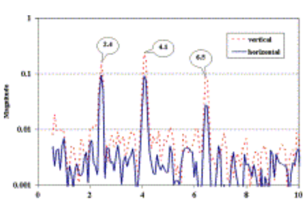


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Fig. 3.

The frequency transform of ground surface vertical and horizontal acceleration.

The accelerometers used in the test are made of piezoresistance, and have the capacity of 2, 5, and 10 g and will cover the frequency range of 0–400 Hz. All the accelerometers are uniaxial, with precision of 0.25%, and are manufactured by ICSensors Co. The 34 accelerometers used in the test include 14, 16, and 4 sensors of types 2, 5, and 10 g, respectively. In order to install the accelerometers, first the steel surface is cleaned and the accelerometer support is installed by using special glue, and the accelerometer is installed on the support, and connected to the dynamic data logger by 50-m cables. Totally 30 accelerometers were installed on the instrumented internal and external beams in the five sections of the bridge. Besides, an accelerometer was used to measure the longitudinal acceleration of the bridge at location of the first section, and three accelerometers were installed on the ground in longitudinal, transversal and vertical directions to measure the acceleration transmitted to the ground due to the passing traffic.

Data collecting and recording were performed by electrical data recording tools. A 48-channel recorder with speed of 2000 datum/s/channel is used for accelerometers dynamic data collecting.

4. Modal identification by fuzzy clustering

4.1. Natural frequency identification

In this section, the bridge deck vibrations are analyzed by processing the acceleration signals. The free vibration part of the deck acceleration signal after passing of the truck with speed of 25 km/h is processed by non-parametric identification method in frequency domain (Bendat & Piersol, 1971). Magnitudes of the free vibration part of the deck acceleration frequency transform in range of 1–10 Hz are presented in Fig. 3,

Fig. 4 and Fig. 5. Fig. 3 depicts the frequency transform of ground surface vertical and horizontal acceleration due to the vibrations of the bridge. As it can be seen, frequencies of 2.4, 4.1, and 6.5 Hz are transmitted to the ground through the deck bearings. Transmission of the vibrations to the adjacent buildings has caused disruption to the inhabitants of the buildings. Fig. 4 and Fig. 5 depict the frequency transform magnitude of beams 1 and 4 vertical accelerations in section 1 (roller bearing) and section 4 (middle of 45-m span). As it can be seen, the whole deck and even the bridge bearings are vibrating with 2.4, 4.1, and 6.5 frequencies as the vibration level in section 1 (roller bearing, location of expansion joint) is higher than the vibration level in middle of the span. Fig. 5 depicts the frequency transform magnitude of beams 1 and 4 vertical accelerations in section 4. The frequencies of 2.2 and 3.6 Hz can be considered as the first (symmetrical) and the second (asymmetrical) frequencies of the bridge. The mentioned frequencies do not have peak magnitudes on the bearings due to formation of nodes. The first and second frequencies of a continuous three-span bridge 30 + 45 + 30 according to reference (Fryba, 1996) is calculated as 2 and 3.9 Hz, which are comparable with the measured magnitudes.

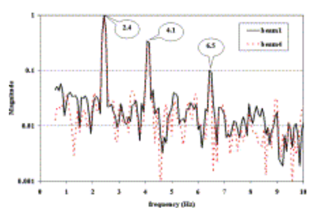

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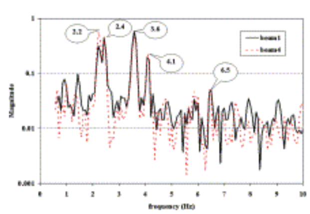
Fig. 4.

The frequency transforms of beams 1 and 4 free vibration vertical acceleration in section 1 (roller bearing).



Full-size image (36K)

Fig. 5.

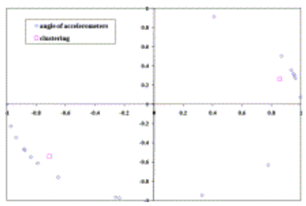
The frequency transforms of beams 1 and 4 free vibration vertical acceleration in section 4 (middle of 45-m span).

4.2. Modal shape identification using fuzzy clustering

The deck acceleration signal frequency function phase should be plotted in each frequency in order to determine the modal shape of the deck vibration in each frequency. The acceleration signal frequency function phase difference of two points in each frequency displays the difference of the two points in reaching the maximum wave magnitude in that frequency. If the difference is 0°, the two points will reach their maximum magnitude simultaneously, and if it is 180°, when one of them reaches its maximum magnitude, the other will reach its minimum magnitude. When the phase difference is 0° or 180°, the modal shape will be a real number, and it means that a stationary wave is generated in the structure. When the phase difference is not 0° or 180°, the modal shape will be complex, and it means that a moving wave is generated in the structure.

The complex mode may occur due to different reasons, practically. The modal shapes resulted from the measurements performed on the structure in operation are often complex. Of course, the mentioned modal shapes are not natural modal shapes in free vibration mode. The complex natural modal shapes are observed in structures with rotating members and gyroscopic forces. But the natural modes in ordinary structures without rotating members will not be complex unless when damping is non-proportional. The non -proportional damping can be expected in practice, since although the internal damping (hysteresis) is distributed proportional to stiffness in most of the structures, but the main part of damping in real structures are caused in members joints which will lead to non-proportional distribution of damping. Hence, this non-proportionality exists in most of the structures. It should be noted that the non-proportional damping is a necessary condition for existence of complex mode and it is not a sufficient condition, at least when the degree of complexity is high. The other necessary condition for existence of complex mode is that two or more modes of the structure are close together. The modes being close together means that the distance of natural frequencies is lower than the governing damping in one or both modes (Ewins, 2000).

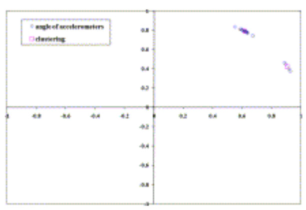
In order to determine the modal shape of accelerometers in each frequency, considering the 30 accelerometer sensors measuring the bridge deck acceleration, fuzzy clustering is used to determine the simultaneous phase difference of accelerometers. It means that the structure accelerometers Fourier transform phase are divided in two clusters in each frequency. If the clustering target function is minimized (accelerometers Fourier transform phases are concentrated in two points), and the phase difference between two representatives of the clusters is 0° or 180°, the real modal shape exists. Otherwise, if the phase dispersal is high or the phase difference between two representatives of the clusters is not 0° or 180°, the complex modal shape exists. The higher the amount of dispersal, or the greater the phase difference of two cluster representatives from 0 and 180, the more complex the modal shape is. The Fourier transform phase clustering of deck vertical accelerometers in frequencies of 2.2 and 2.4 Hz are depicted in Fig. 6 and Fig. 7.



Full-size image (16K)

Fig. 6.

The Fourier transform phase clustering of deck vertical accelerometers in frequency of 2.2 Hz (phase difference of 180° between cluster representatives).



Full-size image (14K)

Fig. 7.

The Fourier transform phase clustering of deck vertical accelerometers in frequency of 2.4 Hz (phase difference of 28° between cluster representatives).

By fuzzy clustering the Fourier transform phase of deck vertical accelerometers, the modal shapes of 2.2, 2.4, and 3.6 Hz frequencies are depicted in Fig. 8, Fig. 9 and Fig. 10. The 2.2 and 3.6 Hz frequencies are the deck frequencies, and the bearings are locations of nodes in these frequencies. But in frequencies of 2.4, 4.1, and 6.5 Hz, the bridge elastic bearings have vibration and the vibrations are transmitted to the ground (Fig. 4).

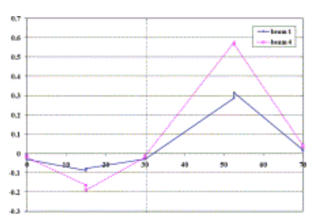


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Fig. 8.

Modal shape of 2.2 Hz frequency.

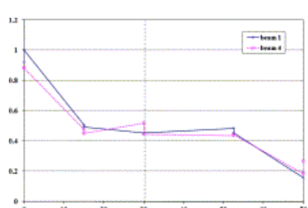

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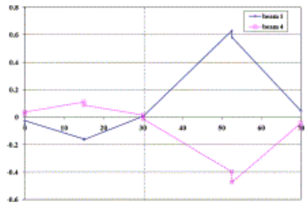
Fig. 9.

Modal shape of 2.4 Hz frequency.



Full-size image (26K)

Fig. 10.

Modal shape of 3.6 Hz frequency.

5. Conclusion

In this article, the non-parametric modal identification algorithm from output measurement has been interpreted corresponding the clustering algorithm by subset relation with complex exponential basis functions on basis of soft computing paradigm. Moreover, the Ghalemorghi Bridge dynamic test under loading of a truck is processed by this identification method, and the acceleration vibration data integration in frequency region (acceleration Fourier transform phase) is performed by fuzzy clustering in order to determine the modal shape.

As it can be observed, we can state that the bridge bearings deformations and vibrations are high, especially in location on of expansion joint. The bridge bearings vibrate with 2.4, 4.1, and 6.5 Hz frequencies (natural frequency of “Deck-Elastic bearing” system). The vibrations are transmitted to the adjacent buildings through the ground and causes disruption to the inhabitants of the buildings. The similarity of the “Deck-Elastic bearing” system to the vibration frequency of deck natural frequency (with rigid bearings), will cause more excitation to the natural frequencies of “Deck-Elastic bearing” system and therefore it leads to bearing vibration. Hence, the Ghalemorghi Bridge major problem is its bearings and this problem can be solved by using non-elastic bearings and installation of appropriate details in location of expansion joint.

Acknowledgments

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